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Question Paper Code : 47227

M.E./M.Tech. DEGREE EXAMINATION, JANUARY 2018
First Semester
Communication Systems
MA5154 – APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS
(Common to M.E. Communication and Networking)
(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. Show that $(1\ 0\ 0)^T$ is a generalized eigen vector of rank 2 corresponding to the eigen value $\lambda = 3$ for the matrix $A = \begin{bmatrix} -7 & -25 & 1 \\ 4 & 13 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
2. Define Toeplitz matrix and give an example of a Toeplitz matrix.
3. When degeneracy occur in transportation problem ? How it can be rectified ?
4. A person requires 10, 12 and12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. If the liquid product sells for Rs. 30 per jar and the dry product sells for Rs. 20 per carton, formulate the linear programming problem in order to minimize the purchase cost and meet the requirement.

5. Write the finite difference scheme of the differential equation $y'' = x + y$ if $h = \frac{1}{4}$.
6. Using point collocation method to solve $y'' + 50 = 0$ where $0 \le x \le 10$.
7. Give any two properties of correlation coefficient.
8. If the joint pdf of (X, Y) is given by $f(x, y) = 2$, in $0 \le x < y \le 1$, find E(X).
9. Write down the Little's formulas that hold good for the infinite capacity single server Poisson queue models.
10. Explain the behaviour of customer in the queuing system.

PART – B (5×13=65 Marks)

11. a) i) Find the singular value decomposition of $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$. (8)

ii) Find the generalized inverse of $\begin{bmatrix} -3 & 1 \\ -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$ (5)

(OR)

b) Find the QR factorization of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$. (13)

12. a) Use simplex method to Maximize $z = x_1 - x_2 + 3x_3$ subject to the constraints : $x_1 + x_2 + x_3 \leq 10$, $2x_1 - x_3 \leq 2$, $2x_1 - 2x_2 + 3x_3 \leq 0$, $x_1, x_2, x_3 \geq 0$. (13)

(OR)

b) i) Use penalty for Big M method to Maximize $z = 3x_1 + 2x_2$ subject to the constraints : $2x_1 + x_2 \leq 2$, $3x_1 + 4x_2 \geq 12$, $x_1, x_2 \geq 0$. (5)

ii) A car hire company has one car at each of five depots P, Q, R, S and T. A customer in each of the five towns, A, B, C, D and E requires a car. The distance in km between the depots (origins) and the towns (destinations) where the customers are, is given in the following distance matrix :

	P	Q	R	S	T
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How should the cars be assigned to the customers so as to minimize the distance travelled ? (8)

13. a) Find y(0.1), y(0.2) from $\frac{dy}{dx} = xy + y^2$, y(0) = 1 by using Runge-kutta method of fourth order. **(13)**

(OR)

b) i) Solve $5x\frac{dy}{dx} + y^2 = 2$, y(4) = 1 at x = 4.4 using Adam Bashforth's predictor – corrector method, given that y(4.1) = 1.0049, y(4.2) = 1.0097 y(4.3) = 1.0143. **(8)**

ii) Using finite difference method, solve $y'' + y + 1 = 0$ in (0, 1) given y(0) = y (1) = 0 taking $h = \frac{1}{4}$. **(5)**

14. a) i) The joint probability mass function of (X, Y) is given by p(x, y) = k(2x + 3y), x = 0, 1, 2 ; y = 1, 2, 3. Find all the marginal and conditional distributions. **(7)**

ii) The joint p.d.f. of X and Y is given by $f(x, y) = e^{-(x+y)}$, x > 0, y > 0. Find the p.d.f. of $\frac{X+Y}{2}$. **(6)**

(OR)

b) i) Two random variables have the joint p.d.f. $f(x, y) = \begin{cases} e^{-y}, 0 < x < y < \infty \\ 0, \quad \text{otherwise} \end{cases}$. Find the regression curves for the means. **(8)**

ii) Two random variables X and Y have the joint p.d.f.

$$f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y), 0 < x < 2, \ 2 < y < 4 \\ 0, \quad \text{otherwise} \end{cases}$$

Find P(X + Y) < 3. **(5)**

15. a) The arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes.

1) Find the average number of persons waiting in the system.
2) What is the probability that a person arriving at the booth will have to wait in the queue ?
3) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call ?
4) Estimate the fraction of the day when the phone will be in use.

5) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for atleast 3 minutes for phone. By how much the flow of arrivals should increase in order to justify a second booth ?

6) What is the average length of the queue that forms from time to time ? **(13)**

(OR)

b) The local one-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour (Exponential service time).

1) What percentage of time is the barber idle ?

2) What fraction of the potential customers are turned away ?

3) What is the expected number of customers waiting for a hair cut ?

4) How much time can a customer expect to spend in the barber shop ? **(13)**

PART – C **(1×15=15 Marks)**

16. a) Food packets have to be air lifted by three aircrafts from an airport and air dropped to five villages. The quantities (suitable units) that can be carried in one trip by these aircrafts to the villages are given below. The total number of trips per day that an aircraft can make to the villages are also given. Find the number of trips each aircraft should make each village so that the total quantity of food transported is maximum : **(15)**

	V1	V2	V3	V4	V5	Trips / day by aircrafts
A1	10	8	6	9	12	50
A2	5	3	8	4	10	90
A3	7	9	6	10	4	60
Trips / day to village	100	80	70	40	20	

(OR)

b) i) Two random variables have joint p.d.f. $f(x, y) = \frac{1}{3}(x + y)$, $0 \le x \le 1$, $0 \le y \le 2$. Find the correlation coefficient. **(10)**

ii) The changes of X, Y, Z becoming managers of a certain company are 4 : 2 : 3. The probabilities that bonus scheme will be introduced if X, Y, Z become managers are 0.3, 0.5 and 0.8 respectively. If the bonus scheme has been introduced, what is the probability that X is appointed as the manager ? **(5)**