Question Paper Code: 54016

B.E./B.Tech. DEGREE EXAMINATION, JANUARY 2018
First Semester
Civil Engineering
MA 8151 – ENGINEERING MATHEMATICS – I
Common to All Branches (Except Marine Engineering)
(Regulations 2017)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Sketch the graph of the function \( f(x) = \begin{cases} 
1 + x & ; x < -1 \\
\frac{x^2}{2} & ; -1 \leq x \leq 1 \\
2 - x & ; x \geq 1 
\end{cases} \)
and use it to determine the value of “a” for which \( \lim_{x \to a} f(x) \) exists?

2. Does the curve \( y = x^4 - 2x^2 + 2 \) have any horizontal tangents? If so where?

3. If \( x = r \cos \theta \) and \( y = r \sin \theta \) then find \( \frac{\partial r}{\partial x} \).

4. If \( x = u \) and \( y = \frac{u}{v} \) then find \( \frac{\partial(x,y)}{\partial(u,v)} \).

5. What is wrong with the equation \( \int_{-1}^{2} \frac{4}{x^3} \, dx = \left[ \frac{-2}{x^2} \right]_{-1}^{2} = \frac{3}{2} \)?

6. Evaluate \( \int_{4}^{\infty} \frac{1}{\sqrt{x}} \, dx \) and determine whether it is convergent or divergent.
7. Find the value of \( \int_0^y \int_0^x \left( \frac{e^{-y}}{y} \right) dx \ dy \).

8. Find the limits of integration in the double integral \( \iint_R f(x, y) \ dx \ dy \) where \( R \) is in the first quadrant and bounded \( x = 1, y = 0, y^2 = 4x \).

9. Convert \( x^2 y'' - 2xy' + 2y = 0 \) into a linear differential equation with constant coefficients.

10. Find the particular integral of \( (D - 1)^2 y = e^x \sin x \).

PART - B  

(5×16=80 Marks)

11. a) i) For what value of the constant "c" is the function "f" continuous on

\[
( - \infty, \infty ), f(x) = \begin{cases} 
  cx^2 + 2x ; & x < 2 \\
  x^3 - cx ; & x \geq 2 
\end{cases}
\]

ii) Find the local maximum and minimum values of \( f(x) = \sqrt{x} - \frac{1}{\sqrt{x}} \) using both the first and second derivative tests.

(OR)

b) i) Find \( y'' \) if \( x^4 + y^4 = 16 \).

ii) Find the tangent line to the equation \( x^3 + y^3 = 6xy \) at the point \((3, 3)\) and at what point the tangent line horizontal in the first quadrant.

12. a) i) If \( u = \left( x^2 + y^2 + z^2 \right)^{-\frac{1}{2}} \) then find the value of \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \).

ii) Find the dimensions of the rectangular box without a top of maximum capacity, whose surface area is 108 sq.cm.

(OR)

b) i) Obtain the Taylor's series expansion of \( x^3 + y^3 + xy^2 \) in terms of powers of \( (x - 1) \) and \( (y - 2) \) up to third degree terms.

ii) Find the maximum or minimum values of \( f(x, y) = 3x^2 - y^2 + x^3 \).
13. a) i) Evaluate \( \int \frac{\tan x}{\sec x + \cos x} \, dx \).  

\[ (8) \]

ii) Evaluate \( \int e^{ax} \cos bx \, dx \) using integration by parts.  

(OR)  

b) i) Evaluate \( \int \frac{x}{\sqrt{x^2 + x + 1}} \, dx \).  

\[ (8) \]

ii) Evaluate \( \int_0^\pi \cos^5 x \, dx \).  

\[ (8) \]

14. a) i) Change the order of integration for the given integral \( \int_0^a \int_0^x (x^2) \, dy \, dx \) and evaluate it.  

\[ (8) \]

ii) Evaluate by changing to polar coordinates \( \iint_0^{2\pi} \int_0^a \frac{x}{\sqrt{x^2 + y^2}} \, dx \, dy \).  

(OR)  

b) i) Evaluate \( \iiint (x \, y \, z) \, dx \, dy \, dz \) over the first octant of \( x^2 + y^2 + z^2 = a^2 \).  

\[ (8) \]

ii) Using double integral, find the area bounded by \( y = x \) and \( y = x^2 \).  

\[ (8) \]

15. a) i) Solve \( \frac{d^2 y}{dx^2} + y = \cot x \) by using method of variation of parameters.  

\[ (8) \]

ii) Solve \( (D^2 - 2D) y = 5e^x \cos x \) by using method of undetermined coefficients.  

(OR)  

b) i) Solve \([(x + 1)^2 D^2 + (x + 1) D + 1] y = 4 \cos \log (x + 1) \).  

\[ (8) \]

ii) Solve \( Dx + y = \sin 2t \) and \( -x + Dy = \cos 2t \).  

\[ (8) \]